

# Nonlinear Power Amplifier Digital Pre-Distortion: A Fast Algorithm

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## ABSTRACT:

The article introduces a quick iterative approach that can pre-equalize a nonlinear dynamic system that has memory. The algorithm's foundation is the secant root-finding approach. It only takes a few rounds to adequately compensate for the system's nonlinear and dynamic impacts, which may be described using tools like the Volterra or Wiener models. Utilizing the procedure, ITERATIVE EQUALIZATION WITH THE SECANT METHOD is subsequently linearized. An illustration of the equalization issue may be seen in Figure 1. The nonlinear dynamic system  $N$  is equalized by a pre-equalizer  $P$ . A Watt amplifier that is not linear and is represented by a Wiener system. The results of the measurements show that the procedure that was given works quite well.  $(u[n] \rightarrow z[n])$

## INTRODUCTION:

For example, power amplifiers in wireless communications are pushed to saturation owing to efficiency reasons, and loudspeakers are nonlinear systems. It is possible to linearize the whole signal channel by pre-equalizing such (often weakly) nonlinear systems. One method that shows promise for pre-compensating nonlinear systems is digital pre-distortion. The basic idea is to digitally pre-distort the signal so that the post-distortion unit and the nonlinear system may interact in a linear fashion. The digital nature of pre-distortion makes it a natural fit for Software Defined Radio (SDR) setups. It is possible to drastically reduce in-band aberrations and unwanted spectral broadening in wireless communication systems.

The increasing signal bandwidths in 3G communication networks cause dynamic effects, also known as memory effects, to be noticeable [1]. Because a nonlinear dynamic system must be pre-equalized, this complicates the pre-distortion operation. The tremendous complexity makes it impossible to use methods based on look-up-tables, which are the tabular inverse of the nonlinear system. Additionally, in the majority of instances, analytical solutions to the pre-inverse of the nonlinear system are unknown. An approximate solution to the pre-distortion challenge is presented in this research by an iterative technique that is based on the secant root-finding method. A microwave power amplifier is used to assess the method's efficacy. Refer to Section 4.1 for further information on how the nonlinear distortion may be significantly reduced.

Figure 1: Nonlinear equalisation problem

Mathematically, the problem can be formulated as

$$N(P(u[n])) = L(u[n]) , \quad (1)$$

where  $L$  is a linear operator and  $u[n]$  denotes the input signal. If  $L = I$ , the identity operator, the filter  $P$  is the pre-inverse of the nonlinear system. Even for static nonlinearities, e.g., polynomials, analytic solutions for the pre-inverse are not known in the majority of cases. If the nonlinear system is dynamic, the inverse is even harder to find. Typical examples of such systems are the often used Volterra series [2, 3] and simplifications, such as Wiener systems (linear filter in front of a static nonlinearity) or Hammerstein systems (static non-linearity in front of a linear filter). Since analytic solutions for the pre-filter  $P$  are seldom known, an iterative method is proposed here which converges very fast, within four to six iterations for the investigated cases. Other approaches exist which determine the parameters of a certain pre-filter  $P$ , e.g., the inverse modelling approach [4]. In this case, the post-inverse is approximated and then used as a pre-

inverse. Depending on the nonlinear filter  $N$  this might give poor results.  
Reformulation of (1) gives

$$N(y[n]) - L(u[n]) = S(y[n]) = 0, \quad (2)$$

with  $P(u[n]) = y[n]$ . The solution to the above equation is the signal after the pre-filter for a specific nonlinear filter  $N$  and a targeted linear operator  $L$ , which in the pre-distortion context is a simple linear amplification,  $L(u[n]) = g \cdot u[n]$ . A fast iterative method for solving (2) for the unknown signal

$y[n]$  (which is equivalent to determining the pre-filter  $P$ ) is the Newton method [5]

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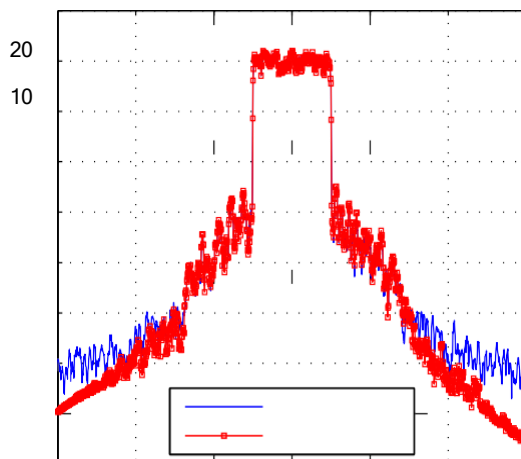
While not very challenging, method (5) does not encounter convergence issues, and three or four iterations are plenty for real-world applications.

A different strategy is sequential approximation, which was first suggested in [7] for generic Volterra systems and then used in [8] for memoryless systems, with (1) rewritten as a fixed-point equation. This approach is much slower than (5) since it converges with linear order; for a comparison with the Newton technique, see [6].

## 2. Black-box model derivation

A Volterra model and a Wiener model, two mathematical black-box models [9], are used to simulate a system. It is used with polynomials up to the sixth order, see (8). With kernel memory lengths  $N_7 = [5, 3, 2, 1]T$ , the Volterra model incorporates all nonlinear components up to the seventh degree. Raising the length of the memory does not lessen the modeling mistake. As saturation levels rise, a decline in the quality of the models becomes apparent. Despite requiring just eight parameters instead of 36 parameters, the Wiener model outperforms the Volterra model in terms of modeling error, which is only marginally greater (max. 1.6 dB). Consequently, the method uses the Wiener model to compute the signal after the pre-distortion filter (5).

Tab. 1 displays the observed spectra of the Wiener model output signal and the system output signal for case 1, whereas Fig. 3 displays the same data. With  $f_s = 10,24$  MHz and the power normalized to 1 W, we can see the results. The third-order nonlinear distortion is clearly predominating, with the fifth-order intermodulation product distortion being almost 50 dB below the in-band signal. The third-order intermodulation products are appropriately calculated, but the fifth- and seventh-order products are under-estimated because of the very weak signal (more than 50 dB weaker than the in-band signal).



Here,  $P_{3,OBO}$  is the signal power in the out-of-band third order harmonic zone (see Fig. 5 for a detailed view) with out-put back-off, whereas  $P_{3,PD}$  is the power in this zone with pre-distortion (PD). Therefore,  $g_3$  denotes the gain by using pre-distortion vs. a simple back-off. The out-of-band third order zone extends over the intervals  $I_{L,3} = [-0, 15; -0, 05]$

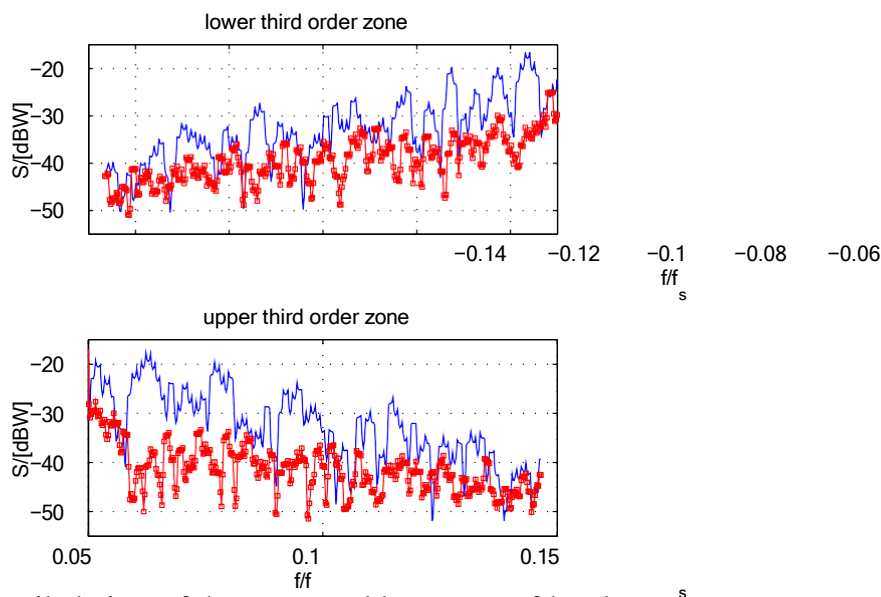
and  $I_{U,3} = [0, 05; 0, 15]$  of the normalised frequency. It has

to be noted that in this zone also higher order intermodulation products are present. The power spectral density in 90% of this zone, resulting in a 5% guard interval to the neighbouring zone, is accumulated, giving the signal power in this zone. In the out-of-band fifth order zone, which extends over the intervals  $I_{L,5} = [-0, 25; -0, 15]$  and  $I_{U,5} = [0, 15; 0, 25]$

of the normalised frequency  $f/f_s$ , no reduction of the signal power could be achieved. From the Figs. 3 and 4 it can be seen that the distortion due to the third order intermodulation products is dominating, the distortion due to higher order intermodulation products is very low, more than 50 dB smaller than the in-band signal. The modelling of this higher order (fifth and seventh) intermodulation products is not accurate enough to achieve a performance gain with pre-distortion reduction in nonlinear distortion compared with the simple back-off becomes smaller, but is still present. The equivalent backing-off is larger in these cases, corresponding to a larger gain-decrease when pre-distortion is used.

The authors have developed an easy-to-understand iterative approach that can pre-equalize nonlinear dynamic systems quickly. We used the algorithm to pre-distort a nonlinear power amplifier. In order to do this, a microwave power amplifier-chain black-box model was developed. It was shown that the power amplifiers under study may function adequately using a quite basic Wiener model. In a comparison with a Volterra model, the Wiener model produced marginally superior modeling outcomes.

Figure 4 shows the spectra of the observed output signal for case 1 with and without pre-distortion (PD), according to Tab. 2. The power amplifier was modelled using a Wiener algorithm.



Detailed view of the upper and lower out-of-band third order harmonic zone for case 1, cf. Tab. 2. The

square markers denote the measured output signal with pre- distortion.

The measured output power, as well as the reduction of the power in the out-of-band third order harmonic zone,  $g_3$ , and the reduction of the overall distortion,  $g_{MSE}$ , compared with backing-off, can be found in Tab. 2. In case 1, which corresponds to the least output power and therefore to the case where the power amplifier is not too much in saturation, pre-distortion yields a significant gain with respect to simple back-off. The spectral broadening due to the third order intermodulation could be reduced significantly, as well as the total distortion, both compared with an equivalent back-off. If the power amplifier is driven by a higher input power, the but required far more parameters.

## CONCLUSION:

Digital pre-distortion, based on the presented method, was then compared to simple back-off, i.e., a



reduction of the input power, resulting in a more linear behaviour of the power amplifier. Measurement results showed that pre-distortion can yield a significant reduction in nonlinear distortion with respect to a back-off. Large gains are achieved if the power amplifier is not driven too much into saturation. For higher saturation levels the linearization results in a significant gain-reduction and the gain of pre-distortion, compared to an equivalent back-off, decreases.

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